

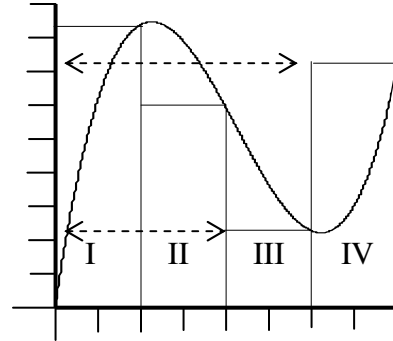
1. (C) To estimate the area under the curve, find the area of the four rectangles shown. The area of a rectangle is the length (also referred to as the height) multiplied by the width (base). Note that the height of rectangles I, III and IV is a fraction above the tick mark representing a whole unit. Because only an approximation is required, the height will be rounded to the nearest half unit.

Area of Rectangle I: $2 \times 8.5 = 17$ square units

Area of Rectangle II: $2 \times 6 = 12$ square units

Area of Rectangle III: $2 \times 2 = 4$ square units

Area of Rectangle IV: $2 \times 7 = 14$ square units



The sum of the areas of these four rectangles is 47 square units which is approximately 50 square units. Choice C is the appropriate answer.

2. (A) Compare the area of the four rectangles to the area under the curve. Rectangles I and IV include more area than the area actually under the curve. The area under the curve is a little more than half the area of these rectangles, producing an over-estimation of about 13 square units. This numerical estimate is based on the area of a triangle: $\frac{1}{2}bh$. Rectangles II and III do not include all the area under the curve; approximately one-fourth of the area is not included in Rectangle II (about 3 square units) and approximately 4 square units is not included in Rectangle III.

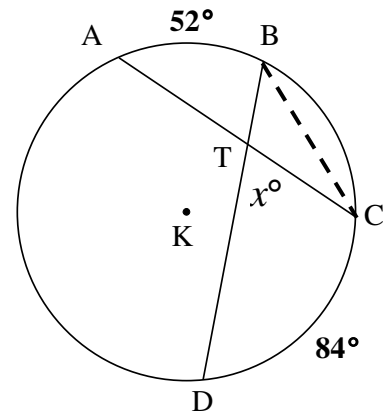
With an over-estimation of approximately 13 square units and an under-estimation of approximately 7 square units, this estimate based on the area of rectangles is greater than the actual area under the curve.

3. (C) Construct segment BC, producing inscribed angles DBC and ACB. Because an inscribed angle equals half the measure of the intercepted arc, the measure of $\angle ACB = \frac{1}{2}(52^\circ) = 26^\circ$ and the measure of $\angle DBC = \frac{1}{2}(84^\circ) = 42^\circ$.

The construction of segment BC also produced $\triangle BCT$. Because the sum of the angles of a triangle is 180° , $\angle BTC = 180 - (26 + 42) = 180 - 68 = 112^\circ$.

$\angle BTC$ and x are a linear pair and therefore supplementary. In other words, the sum of these two angles is 180° . If $\angle BTC = 112$, then $x = 180 - 112 = 68^\circ$.

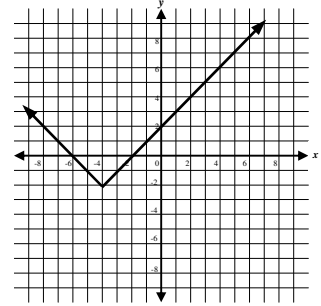
[Note that the exterior angle of the triangle (x) is equal to the sum of the two remote interior angles, $\angle B + \angle C$.]



[Additional Note to Geometry students: Recall that an angle formed by two chords is equal to half the sum of the intercepted arcs: $\frac{1}{2}(52 + 84) = 68^\circ$]

1. (D) Counting the number of gypsy moth caterpillars on one tree before the spraying and again one month after the spraying would be difficult and not very reliable. Checking under every leaf on even a small tree might not produce an accurate count and caterpillars can change location. Choice B would also be difficult and unreliable, assuming that the bodies of dead caterpillars would remain to be counted at the end of one month. Choice C, examining photographs for defoliation, would be even less valid. Although Choice D has several inherent flaws, it does produce a controlled sample from the population which can be accurately counted before and after the spray.

2. (D) If the graph of the function $f(x) = |x|$ is translated 4 units to the left and 2 units down (as shown on the graph to the right), the vertex of this new function is the point $(-4, -2)$. To identify the correct function for this new graph, determine which of the four choices has a function value of -2 when x equals -4 . [The y -value of the ordered pair represents the function value.] Substitute -4 for x and simplify.



Choice A: $f(x) = |x - 2| + 4$

Substitute -4 for x : $f(-4) = |-4 - 2| + 4$

Simplify: $f(-4) = |-6| + 4 = 6 + 4 = 10$

$f(-4) = 10 \neq -2$

Choice B: $f(x) = |x + 2| - 4$

$f(-4) = |-4 + 2| - 4$

$f(-4) = |-2| - 4 = 2 - 4 = -2$

$f(-4) = -2$

Choice C: $f(x) = |x - 4| + 2$

Substitute -4 for x : $f(-4) = |-4 - 4| + 2$

Simplify: $f(-4) = |-8| + 2 = 8 + 2 = 10$

$f(-4) = 10 \neq -2$

Choice D: $f(x) = |x + 4| - 2$

$f(-4) = |-4 + 4| - 2$

$f(-4) = |0| - 2 = 0 - 2 = -2$

$f(-4) = -2$

Because the functions in choices B and D both contain the point $(-4, -2)$, a second point must be selected. From the graph, note that the y -intercept is $(0, 2)$. To determine which of the two choices has a function value of 2 , substitute 0 for x into these two functions and simplify.

Choice B: $f(x) = |x + 2| - 4$

Substitute 0 for x : $f(0) = |0 + 2| - 4$

Simplify: $f(0) = |2| - 4 = 2 - 4 = -2$

Choice D: $f(x) = |x + 4| - 2$

$f(0) = |0 + 4| - 2$

$f(0) = |4| - 2 = 4 - 2 = 2$

Since choice D has a function value of 2 when $x = 0$, it is the correct answer.

3. (D) Simplify using the order of operations.

Parenteses (Multiply before adding): $6 - (4 + 5 \cdot 2) \div (2 - 3)^3 =$

Complete Parenteses: $6 - (4 + 10) \div (-1)^3 =$

Exponents: $6 - 14 \div (-1)^3 =$

Divide $(-14 \div -1)$: $6 - 14 \div -1 =$

Add: $6 + 14 =$

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1. **(D)** Choice A is incorrect because most 9th grade students are not athletes. If 150 are involved in sports, then 390 are not involved in sports. Choice B may or may not be true. However there is not enough information given to draw that conclusion. Choice C is also incorrect. Although less than one third of 9th grade students are athletes, there is no information given concerning students in grades 10, 11 or 12. Choice D is the best answer because more than two thirds of those involved in extra-curricular activities are involved in sports which makes this statement a reasonable conclusion.
2. **(D)** Since a percent is a fractional part of 100, 56% really represents 56 parts out of 100. This can be written as the fraction $\frac{56}{100}$. To reduce the fraction to simplest form, divide both numerator (56) and denominator (100) by common factors until 1 is the only factor they share. 56 and 100 are both divisible by 4, so divide both to reduce the fraction to $\frac{14}{25}$. 14 and 25 only share a factor of 1, so the fraction $\frac{14}{25}$ is in simplest form.
3. **(C)** Although many algebra students are capable of solving this rational equation by multiplying both sides by the least common denominator, it may be easier to substitute each of the values given into the equation to determine the correct solution.

Choice A ($x = -2$ or $x = 7$) - Substitute -2 for x :
$$\frac{-2}{-2+2} - \frac{10}{(-2)^2 - (-2) - 6} \stackrel{?}{=} \frac{2}{-2-3}$$

Note that the denominator in the first term is 0. Because division by 0 is not possible, -2 cannot be a solution to this equation. It is not necessary to test the second value because Choice A is incorrect.

Choice B ($x = 2$ or $x = -7$) - Substitute 2 for x :
$$\frac{2}{2+2} - \frac{10}{(2)^2 - (2) - 6} \stackrel{?}{=} \frac{2}{2-3}$$

Simplify:
$$\frac{2}{4} - \frac{10}{-4} = \frac{2}{4} + \frac{10}{4} \stackrel{?}{=} \frac{2}{-1}$$

$$\frac{12}{4} = 3 \neq -2$$

Because $x = 2$ does not produce a true statement, it cannot be a solution. It is therefore unnecessary to test the second value.

Choice C ($x = 7$) - Substitute 7 for x :
$$\frac{7}{7+2} - \frac{10}{(7)^2 - (7) - 6} \stackrel{?}{=} \frac{2}{7-3}$$

Simplify:
$$\frac{7}{9} - \frac{10}{36} \stackrel{?}{=} \frac{2}{4}$$

Produce common denominators and combine:
$$\frac{28}{36} - \frac{10}{36} = \frac{18}{36} \stackrel{?}{=} \frac{2}{4}$$

Simplify both fractions:
$$\frac{1}{2} = \frac{1}{2}$$

Because this is a true statement, $x = 7$ is a solution to this rational equation.

1. (A) In order to add or subtract matrices, the orders (dimensions) of the matrices must be the same. Since the order of both matrices is 3×2 (3 rows \times 2 columns), it is possible to subtract the matrices. After subtracting, the order of the resulting matrix will also be 3×2 . To subtract matrices, simply subtract corresponding elements as follows:

$$\begin{bmatrix} 0 & 4 \\ 7 & -5 \\ -6 & -4 \end{bmatrix} - \begin{bmatrix} 9 & -1 \\ 8 & 3 \\ -2 & 10 \end{bmatrix} = \begin{bmatrix} 0-9 & 4--1 \\ 7-8 & -5-3 \\ -6--2 & -4-10 \end{bmatrix} = \begin{bmatrix} -9 & 5 \\ -1 & -8 \\ -4 & -14 \end{bmatrix}$$

The correct matrix is choice A.

2. (B) From the second sentence, it is clear that sashimi is equivalent to raw, slivered fish. The phrase “raw, slivered fish” can replace the word “sashimi” in the first sentence. This produces the conclusion: “Susie loves raw, slivered fish.”
3. (B) The sequence given is the Fibonacci sequence, well known for its appearance in nature. To produce a term, find the sum of the previous two terms. For example, $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$, $3 + 5 = 8$, etc. To find the first twelve terms, continue the pattern to generate all terms up to and including the twelfth:

Term number	Pattern: add two preceding terms	Term
1		1
2		1
3	$1 + 1 = 2$	2
4	$1 + 2 = 3$	3
5	$2 + 3 = 5$	5
6	$3 + 5 = 8$	8
7	$5 + 8 = 13$	13
8	$8 + 13 = 21$	21
9	$13 + 21 = 34$	34
10	$21 + 34 = 55$	55
11	$34 + 55 = 89$	89
12	$55 + 89 = 144$	144

Now to find the sum of the first twelve terms, add all the numbers in the last column of the table. The total is 376.

1. **(D)** Choices A and B would not produce reliable data because he is surveying only teenagers who use the computer. Choice C limits the survey to only those students at one specific high school. Choice D would provide the most reliable data about teenagers in his city and their computer use. Even though Tuesday is a specific day of the week and the cafeteria is a specific place, Tuesday is just one randomly selected day of the week and the cafeteria is a randomly selected location.
2. **(B)** Aaron falls in the same category as the majority of the teenagers surveyed (second from left labeled 2-3 hours). The three categories to the right (4-5 hours, 6-7 hours, and 8-9 hours) represent the number of teenagers who spend more time each day playing games than Aaron. Although there are no numbers of students given, the percentage can be calculated by counting the number of shaded sections. There are a total of 15 shaded sections and 7 shaded sections labeled as more than 3 hours of computer use per day. The percentage is found by comparing the part (7 sections) to the whole (15 sections), converting from fraction to decimal to percent. Approximately 47% of the teenagers surveyed spend more time each day playing games than Aaron.
3. **(D)** The area of the shaded region can be found by taking the difference between the area of the outer region (in this case, the circle) and the area of the inner region (the hexagon). Find the area of the circle by substituting 10 for r into the formula given on the Formula Sheet: $\pi r^2 = \pi(10)^2 = \pi \cdot 100 = 100\pi$.

The dotted lines on the diagram below represent the radii of circle S. Because all radii of a circle are congruent (equal) and the sides of a regular hexagon are congruent, the six triangles that make up the hexagon are also congruent by SSS (side-side-side). Congruent triangles have equal areas. To find the area of the hexagon, find the area of one triangle such as $\triangle STQ$ and multiply by 6. The area of a triangle is $\frac{1}{2}bh$. The height (SP) is given but the length of the base (TQ) must be determined. $\triangle STP \cong \triangle SQP$ by HL (hypotenuse-leg) which means that $TP = PQ$ by CPCTC (corresponding parts of congruent triangles are congruent). Because $\triangle STP$ is a right triangle (see diagram), find TP by using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

Substitute triangle sides: $(TP)^2 + (SP)^2 = (ST)^2$

Substitute lengths: $(TP)^2 + (5\sqrt{3})^2 = 10^2$

Simplify: $(TP)^2 + 75 = 100$

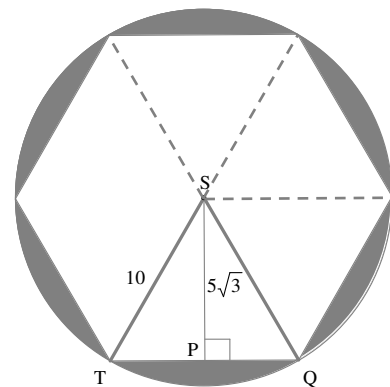
Subtract 75 from both sides: $(TP)^2 = 25$

Take the square root of both sides: $TP = 5$

Note: In Geometry, only the principal square root which is the positive root is needed.

So if $TP = 5$, then $TQ = 10$. The length of the base of the triangle is 10.

The area of $\triangle STQ = \frac{1}{2}bh = \frac{1}{2}(10)(5\sqrt{3}) = 25\sqrt{3}$. Since the area of $\triangle STQ = 25\sqrt{3}$, then the area of the hexagon must be $6 \cdot 25\sqrt{3} = 150\sqrt{3}$. The area of the shaded region is therefore $100\pi - 150\sqrt{3}$.



1. **(D)** The values in the table can be considered ordered pairs, where the first column represents the x -coordinates and the second column represents the y -coordinates. To find the slope of a line, two points (or ordered pairs) are needed. Choose any two, since the slope remains the same between any two points on the line. The first two ordered pairs from the table will be used. Let $(3000, 225,000) = (x_1, y_1)$ and $(5500, 412,500) = (x_2, y_2)$. Use the slope formula given on the Formula Sheet and substitute the appropriate values:

$$\begin{aligned} \text{Slope Formula:} \quad m &= \frac{y_2 - y_1}{x_2 - x_1} \\ \text{Substitute:} \quad m &= \frac{412,500 - 225,000}{5500 - 3000} \\ \text{Simplify:} \quad m &= \frac{187,500}{2500} = 75 \end{aligned}$$

So the slope of the function is 75.

2. **(B)** Slope represents a rate of change between two variables. Examine the slope formula. Note that the numerator is a change in the y -values. In this situation, the y -values represent gallons of water. The denominator is a change in x -values, which represents the number of people.

Looking at the labels when calculating slope, notice that daily water usage (in gallons) is divided by population. This produces a numerical expression involving the following labels:

$$\frac{\text{gallons of water}}{\text{people}}$$

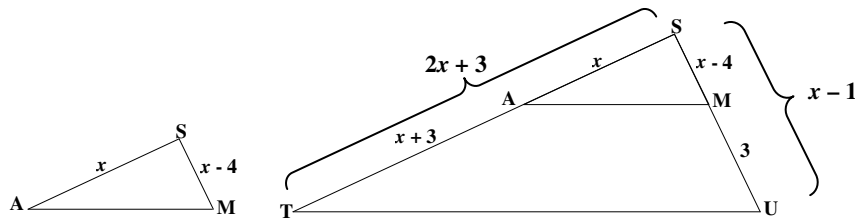
These labels can be rewritten as gallons of water per person. Therefore the slope represents the number of gallons of water per person.

3. **(D)** Since the division bar in the expression $\frac{31 - 7.2}{2\pi}$ indicates a grouping symbol, the expression could be rewritten as $\frac{(31 - 7.2)}{(2\pi)}$. This is also equivalent to $(31 - 7.2) / (2 \cdot \pi)$.

To evaluate this expression using a calculator, enter the characters as they appear from left to right. Be sure to include all of the parentheses and note that “/” indicates division and “•” indicates multiplication. Choice D is the correct answer.

Gaining Math Momentum for the PSSA

- (D)** For an ordered pair to be a solution to a system of inequalities, it must appear within the shaded region of the graph or on a solid line boundary of the shaded region. Check each of the choices to determine whether it is a solution. Choice A (0, -8), on the y -axis, does not fall within the shaded area. Choice B (1, 2), in the first quadrant, falls on the dotted line bordering the shaded region. However points on the dotted line are not part of the solution set, so choice B is not correct. Choice C (-2, -5), in the third quadrant, is located just outside of the shaded area. Choice D (-10, 4) is located in the second quadrant beyond the given graph, but recall that the lines and shading in the coordinate plane all extend infinitely. If the graph were redrawn to include the point (-10, 4), it would fall within the shaded region. Choice D is the correct answer.
- (C)** The maximum point of a graph is vertically the “highest” point, so look for this point on the given graph. It is right in the middle. In this situation, the altitude of the balloon depends on the amount of time that has passed since the balloon left the ground, making altitude the dependent variable (y) and time the independent variable (x). Clearly the x -coordinate of the maximum is 40 minutes. The y -coordinate falls half way between 1000 and 1500 ft. at about 1250 ft. So in this situation, the maximum point (40 min., 1250 ft.) indicates that the balloon reaches its maximum altitude of 1250 feet after 40 minutes.
- (D)** Since $AM \parallel TU$, there are two sets of corresponding angles: $\angle SAM \cong \angle STU$ and $\angle SMA \cong \angle SUT$ (recall corresponding angles of parallel lines are congruent). Therefore $\triangle SAM$ is similar to $\triangle STU$ by AA (angle-angle). Redraw the triangles separately, labeling the lengths of the sides. Add the smaller segments to find the lengths of the sides of the larger triangle.



Since the triangles are similar, the corresponding sides of the triangles are proportional. Write a proportion using the given sides and solve.

$$\frac{x}{2x+3} = \frac{x-4}{x-1}$$

Multiply both sides by the common

denominator $(2x+3)(x-1)$:

Simplify:

$$x(x-1) = (x-4)(2x+3)$$

$$x^2 - x = 2x^2 + 3x - 8x - 12$$

$$x^2 - x = 2x^2 - 5x - 12$$

Subtract x^2 from both sides:

$$-x = x^2 - 5x - 12$$

Add x to both sides:

$$0 = x^2 - 4x - 12$$

Factor:

$$0 = (x-6)(x+2)$$

In order for the product to equal zero, either one or both factors must equal zero. So set each factor equal to zero and solve:

$$x - 6 = 0 \text{ or } x + 2 = 0$$

$$x = 6 \text{ or } x = -2$$

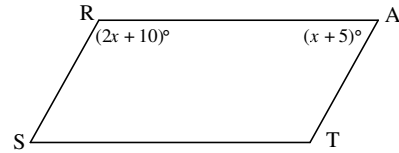
Since x represents a length, $x = -2$ is not a possible solution. Therefore x must equal 6. Now find the length of ST . Using the redrawn triangles, note that ST has a length of $2x + 3$. Substitute 6 for x and simplify: $2(6) + 3 = 12 + 3 = 15$.

Note: For more information on factoring, see solution #3 on page 63.

Gaining Math Momentum for the PSSA

1. **(B)** Because RATS is a parallelogram, opposite sides must be parallel. Recall that interior angles on the same side of a transversal are supplementary if lines are parallel. If segment RS is parallel to segment AT, then $\angle R$ and $\angle A$ are supplementary (sum to 180°). To find the value of x , solve the following equation:

$$\begin{array}{l} \angle R + \angle A = 180 \\ \text{Substitute:} \quad 2x + 10 + x + 5 = 180 \\ \text{Combine like terms:} \quad 3x + 15 = 180 \\ \text{Subtract 15 from both sides:} \quad 3x = 165 \\ \text{Divide both sides by 3:} \quad x = 55 \end{array}$$



To find the measure of $\angle RST$, first find $\angle R$ by substituting the value for x into the expression for $\angle R$.

$$\begin{array}{l} \angle R = 2x + 10 \\ \text{Substitute:} \quad = 2(55) + 10 \\ \text{Simplify:} \quad = 110 + 10 = 120^\circ \end{array}$$

Because segment RA must be parallel to segment ST, $\angle R$ is supplementary to $\angle RST$.

$$\begin{array}{l} \angle R + \angle RST = 180 \\ \text{Substitute:} \quad 120 + \angle RST = 180 \\ \text{Subtract 120 from both sides:} \quad \angle RST = 60^\circ \end{array}$$

Also note that opposite angles of a parallelogram are congruent:

$$\angle A = x + 5 = 55 + 5 = 60^\circ = \angle RST$$

2. **(C)** To translate the phrase “six less than twice some number” into an algebraic expression, replace “some number” with a variable, x . “Twice” means to multiply that number by 2, producing the expression $2x$. “Six less than...” means that 6 is subtracted from $2x$. In other words, “six less than twice some number” means twice some number minus 6 or $2x - 6$. The correct answer is C.
3. **(D)** The ratio of Cheryl’s time to Darryl’s time is:

$$\frac{1.5 \text{ hours}}{20 \text{ minutes}}$$

In order to simplify this ratio, both units must be the same. To change hours to minutes, recall that there are 60 minutes in each hour. Multiply 1.5 hours by 60 minutes per hour which equals 90 minutes.

The ratio 90 minutes:20 minutes simplifies to 9:2. The answer is choice D.